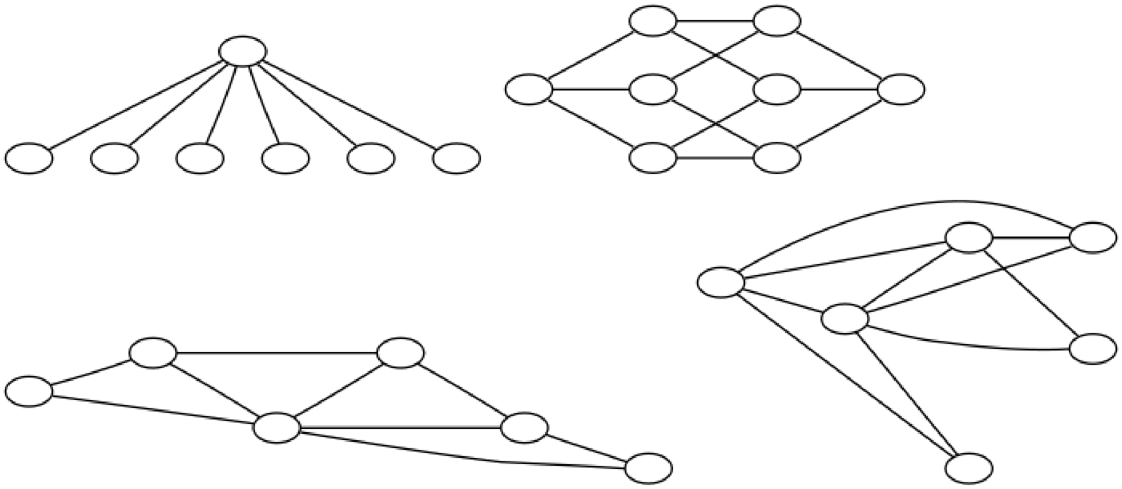
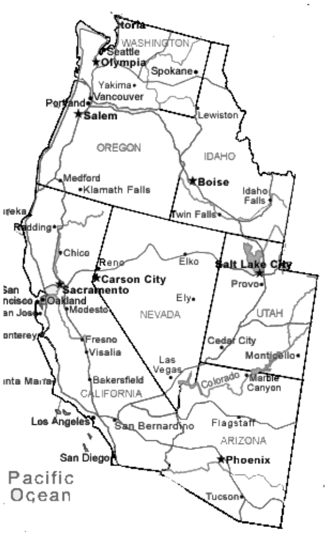
Graph Theory and Data Compression worksheet

1. Color these graphs (or label the vertices with numbers) with the fewest colors possible:



2. Convert these states into a graph where the states are the vertices and the edges indicate two states touch. Color the resulting graph.



3. For the following segments of code, draw vertical bars indicating when the variables are used. Then draw a graph with the variables as vertices and color it to assign them to registers.

```
c = a + b              c = a - b              d = a + b * c
d = c + e + f          d = 2 * c              e = a * b + c - d
f = c - g              e = d + 4              f = (e / 2) - d
h = f - d              f = e / 2              g = f * f
```

4. Can you make a graph that has no clique of size 3 (triangle) but requires 3 colors?

5. **Challenge:** Try to construct a graph with no clique of size 4 that requires 4 colors.

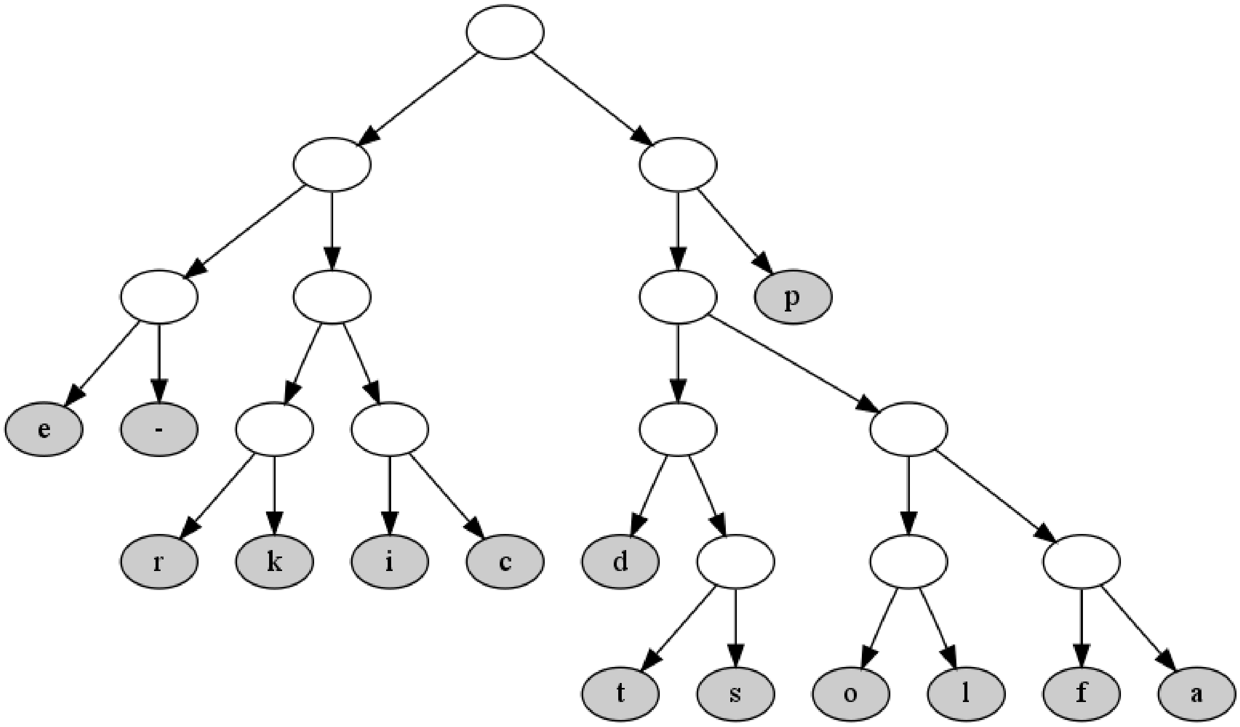
6. **Challenge:** Try to construct a graph with no clique of size 3 (i.e. a triangle) that requires 4 colors. Hint: your graph will need at least 11 vertices.

7. Create a Huffman encoding binary tree to use on the following phrase (including spaces): ”she sells seashells”.

8. Use that tree to encode the message. How many bits does it take?

9. If you had used a standard encoding that takes 3 bits per character, how many bits would that take to encode the message?

10. Decode the following message given the Huffman encoding tree below. (note that the ”-” indicates a space)



11000100100000100001
110110110000100001
11011001110101000100000 1
10111001
101011010010010001
1010010110001
110110011101011010100010000 01
11000111100000010010011

11. **Challenge:** Assume you’re sending a message with the letters ”a” and ”b” where ”a” has probability .9 and ”b” has probability .1. Draw a Huffman encoding tree for this message. What is the average number of bits per character of a message? Try to think of a way to reduce this. Hint: try combining characters!

12. **Challenge:** Here are the 11 most common letters in English along with their relative frequencies:

e	t	a	o	i	n	s	h	r	d	l
16.3%	11.7%	10.5%	9.6%	9.0%	8.6%	8.1%	7.8%	7.7%	5.5%	5.2%

Construct a Huffman encoding tree, and determine the average number of bits per character using this code given the relative frequencies.